GCSE MATHEMATICS

S21-C300

With Calculator Assessment Resource M

Higher Tier

Formula list

Area and volume formulae

Where r is the radius of the sphere or cone, l is the slant height of a cone and h is the perpendicular height of a cone:

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Kinematics formulae

Where a is constant acceleration, u is initial velocity, v is final velocity, s is displacement from the position when $t = 0$ and t is time taken:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

1.

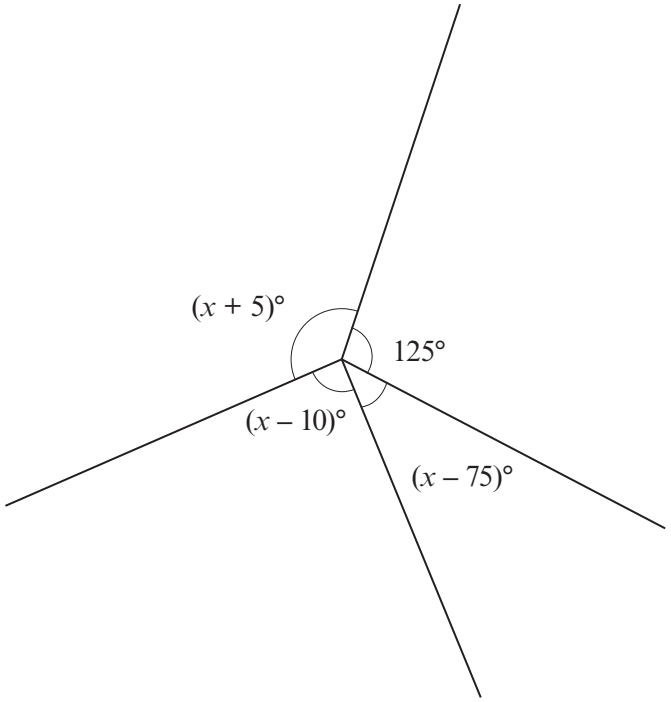


Diagram not drawn to scale

Write an equation in terms of x and solve it.
You must show all your working. [3]

$360 = (x + 5) + (x - 10) + (x - 75) + 125$

$235 = x + 5 + x - 10 + x - 75$

$235 = 3x - 80$

$315 = 3x$

$105^{\circ} = x$

$x =$ 105°

2. *(a)* Expand and simplify $(2x - 7)(3x - 8)$. [3]

$6x^2 - 16x - 21x + 56$

$= 6x^2 - 37x + 56.$

(b) Solve $w^2 + 8w - 33 = 0$. [3]

$(w + 4)^2 - 16 - 33 = 0$

$(w + 4)^2 = 49$

$w + 4 = \pm 7$

$w + 4 = 7$ $\quad$ $w + 4 = -7$

$w = 3$ $\quad$ $w = -11$

11, −3 → 8 sum, −33 product

$(w+11)(w-3) = 0$

$w + 11 = 0$ or $w - 3 = 0$

$w = -11$ or 3

(c) Factorise $b^2 - 144$. [1]

$(b + 12)(b - 12)$

(d) Rearrange the following to make e the subject.
Simplify your answer.

$$9e^2 = t^4$$ [2]

$9e^2 = t^4$

$e^2 = \frac{t^4}{9}$

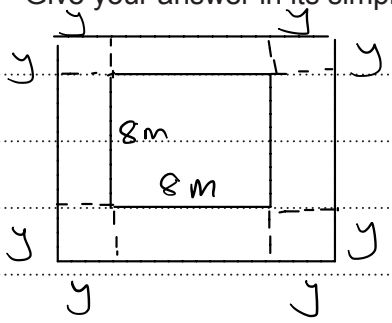
$e = \sqrt{\frac{t^4}{9}} = \frac{t^2}{3}$

$\therefore \quad e = \frac{t^2}{3}$

3. A square lawn has a side of length 8 m.
A builder is asked to make a path around the outside edge of the square lawn.

(a) The plan was for a path of width y metres with all the edges of the path being straight.

Find an expression for the area of this path in terms of y.
Give your answer in its simplest form. [4]



sides = $(8 + 2y)$

area of lawn = $8 \times 8 = 64 m^2$

Total area of lawn =

$(8 + 2y)(8 + 2y)$

$64 + 16y + 16y + 4y^2$

$64 + 32y + 4y^2$

area of path =

$= 4y^2 + 32y + 64 - 64$

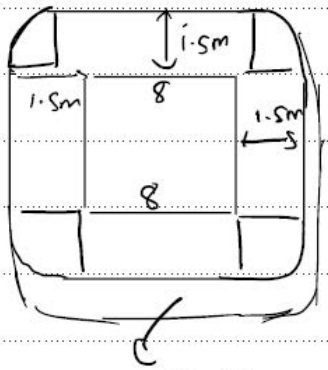
$= 4y^2 + 32y$

$= 4y(y + 8)$

(b) Before the builder started, the plan was updated and the following agreed:

- at each corner of the lawn the path should be a quarter circle,
- the width of the path is to be 1·5 m,
- the path is to be made of 8 cm thick concrete.

Calculate the volume of the concrete needed for the path. [5]



$4 \times \frac{1}{4}$ circle = 1 circle

circle $= \pi r^2 = \pi (1.5)^2$

$= 2.25\pi$

$= 7.0685\ m^2$

8 cm thick = 0.08m

Path rectangles

$4 \times (1.5 \times 8) = 48\ m^2$

Therefore cross-sectional area

$= (48 + 7.068)\ m^2$

8cm → 0.08m

∴ Volume $= (48 + 7.068)\ 0.08$

$= 4.41\ m^3$ (2dp)

4. *(a)* Factorise and hence solve the following equation. [3]

$$4x^2 + 16x + 15 = 0$$

$(2x + 3)(2x + 5)$

$2x + 3 = 0$ $\quad$ $2x + 5 = 0$

$2x = -3$ OR $2x = -5$

$x = -\frac{3}{2}$ $\quad$ $x = -\frac{5}{2}$

(b) Find the nth term of the following sequence. [2]

7, 10, 15, 22, 31, 42, ...

+3 +5 +7 +9 +11

2 2 2 2

1	2	3	...	n
1	4	9		n^2
7	10	15		n^2+6

$\rightarrow n^2 + 6$

(c) Ali walks slowly from home through a forest and back home again.

He walks a total distance of $(3x + 1)$ km in a time of $(\frac{x}{2} + 6)$ hours.
Ali's average speed is 2 km/h.
He left home at 9 a.m.

At what time did Ali return home? [4]

Total distance = $3x + 1$ km

Total time = $\frac{x}{2} + 6$ hours

Avg speed = km/h

Avg speed = $\frac{\text{total distance}}{\text{total time}}$

$2 = \frac{3x + 1}{\frac{x}{2} + 6} = \frac{6x + 2}{x + 12}$

$2x + 24 = 6x + 2$

$22 = 4x$

$x = 5.5$

hours = $\frac{5.5}{2} + 6$

$= 8.75$ hours

9am + 8.75 hours

$= 17:45$

$= 5:45$ pm

Ali returned home at 5.45 pm

(d) The expression $x^2 + 18x + 2$ has a minimum value.
By **completing the square**, complete the statements below.
You must show all your working. [3]

'The minimum value of $x^2 + 18x + 2$ occurs when $x =$ -9'

'The minimum value of $x^2 + 18x + 2$ is -79'

$(x + 9)^2 - 81 + 2$

$(x + 9)^2 - 79$

5. *(a)* The volume of a cone is equal to the volume of a sphere.
The radius of the cone is four times the radius of the sphere.

Show that the perpendicular height of the cone is a quarter of the radius of the sphere. [4]

$\frac{1}{3}\pi r_c^2 h = \frac{4}{3}\pi r_s^3$ | $r_c = 4r_s$

$\pi r_c^2 h = 4\pi r_s^3$

$\pi (4r_s)^2 h = 4\pi r_s^3$

$\pi 16 r_s^2 h = 4\pi r_s^3$

$h = \frac{4\pi r_s^3}{16\pi r_s^2} = \frac{1}{4} r_s^1 = \frac{1}{4} r_s$

$\therefore h = \frac{1}{4} r_s$ as required

(b) The radii of two spheres are in the ratio 2 : 7.
The volume of the smaller sphere is 10·4 cm^3.

Calculate the volume of the larger sphere. [3]

large sphere $r = 7r$

small sphere $r = 2r$

$\frac{4}{3}\pi (7r)^3 : \frac{4}{3}\pi (2r)^3$ | $V_s = 1 \times 10.4 cm^3 = 10.4$

$343r^3 : 8r^3$ | $V_L = \frac{343}{8} \times 10.4 = 445.9$

$343 : 8$

$\frac{343}{8} : 1$ | $V_L = 445.9 cm^3$